Workshop: Innovative Low-Volatility-Strategien

Raul Leote de Carvalho

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The markets through CAPM lenses

In CAPM the market cap is the most diversified portfolio with highest Sharpe ratio

- **CAPM:**
  - Equilibrium model
    - Supply equals demand
  - returns are proportional to beta
    - exposure to specific risk does not pay
  - The market cap portfolio
    - is the most diversified portfolio
    - has the highest Sharpe ratio

- **Minimum variance portfolio**
  - invests in low volatility and uncorrelated stocks
  - has beta < 1, i.e. should under-perform the market cap portfolio
  - is exposed to specific risk, i.e. should have lower Sharpe ratio than the market cap portfolio
Is CAPM enough?

An *incomplete* description of markets

- Systematic deviations to CAPM: *anomalies* or *mis-pricings*
  - Investor preference for some stocks breaks supply-demand equilibrium
  - Mis-pricings could be arbitraged away should investors give up such preferences

- Empirical evidence of *mis-pricings*:
  - 1977: Value anomaly (Basu, *Journal of Finance*)
  - 1979: Earnings Revisions anomaly (Lakonishok and Givoly, *Journal of Accounting and Economics*)
  - 1981: Small Cap stocks anomaly (Banz, *Journal of Financials Economics*)
  - 1990: Short-term reversal anomaly (Jagadeesh, *Journal of Finance*)
  - 1993: Momentum anomaly (Jagadeesh and Titman, *Journal of Finance*)
Risk anomaly

The risk puzzle in equity markets

- Empirical observation in US over 85 years and global markets over 20 years:
  - Relationship risk-return is flatten, even inverted, in equity markets
  - Higher risk-adjusted returns from portfolios invested in low risk stocks
  - Lower risk-adjusted returns from portfolios invested in high risk stocks

- Haugen and Heins showed empirically that in the US, between 1926–1971:
  
  “over the long run stock portfolios with lesser variance in monthly returns have experienced greater average returns then their ‘riskier’ counterparts”

- Examples of other articles confirming this empirical evidence

- Examples of empirical evidence for Global Markets
Risk anomaly
The risk puzzle in equity markets

- Higher demand for risky stocks: bids up prices and lowers future returns
  - Why?

  1. Leverage restrictions
     Black (1993)

  2. Short-selling distortions
     De Giorgi and Post (2011)

  3. Active management objective to outperform market cap indices
     Blitz and van Vliet (2007)
     1. Low risk stocks: higher Sharpe ratio but also large TE and may be even have negative IR

  4. Fund managers compensation creates incentive to invest in risky stocks
     Siri and Tufano (1998)
     1. Asymmetric option like compensation structure

  5. Investor perceiving stocks as lottery tickets
     Barberis and Huang (2008)
Risk anomaly

Theoretical demonstrations of the risk anomaly

- Factors disrupting the supply-demand equilibrium in favour of risky stocks
  - Jensen, Black and Scholes (1972), Miller and Scholes (1972)
    - Investors cannot borrow at risk-free rate
  - Black (1972), Black (1993)
    - Borrowing restrictions and borrowing reluctance provide mechanism for bidding up prices of riskier stocks
  - Frazzini and Pedersen (2010)
    - Some investors are prohibited from using leverage, others limited due to margin requirements
  - Blitz (2012)
    - Benchmarking against the market cap portfolio promotes investment in riskier stocks

- Diversification in market cap portfolio as source of risk anomaly
  - Berrada, Messikh, Oderda, Pictet (2012)
    - Total capitalisation is never concentrated in one single stock
Risk anomaly

Counter-evidence?

- Attempts to provide counter-evidence have failed:
  - Martelini (2008): biased sample includes only surviving stocks
    - Importance of survivorship bias for the anomalous behaviour of risky stocks
  - Fu (2009): look-ahead bias in estimation procedure
    - Several studies have since shown Fu’s results no longer hold after correcting for a look-ahead bias
  - Bali and Cakici (2008): less significance of anomaly in one very specific case
    - Anomaly is less significant when
      - large-caps only
      - risk measure is long-term volatility
      - evaluation measure is 3-factor alpha
      - returns are on a simple (arithmetic) basis
  - Huang et al. (2010): indeed, if short-term volatility is used then there is exposure to short-term reversal
    - Other research shows that the anomaly is robust to the short-term reversal
  - Amenc et al. (2013): just reflects the need to rebalance and remove low risk stocks with increasing volatility
    - Indeed, it is an active strategy, requires rebalancing
First Generation approaches to Low Volatility Equity investing

Risk-based systematic approaches

- Equity risk-based strategies: systematic quantitative approaches to stock allocation which
  - do not require any explicit stock return forecasts
  - rely only on risk models to manage risk
  - translate into portfolios that are periodically rebalanced to take into account drift and changes in risk models.

- Four risk-based strategies that provide exposure to low volatility stocks:
  - Equal Risk Budgeting (ERB): Stock weights are inversely proportional to their volatility
  - Equal Risk Contribution (ECR): Stock weights are chosen so that contribution to risk is the same for all
  - Minimum Variance (MV): Portfolio with the lowest ex-ante volatility
  - Maximum Diversification (MD): Maximises a diversification ratio

First Generation approaches to Low Volatility Equity investing

Application to large universe of stocks

- Back-tests 1995 - 2010
  - No transaction costs

- Non-constrained portfolios
  - MV and MD also as Long-only and maximum weight 5% for each stock

- Quarterly re-balancing
  - Mar, Jun, Sep, Dec

- A Principal Components risk model: 2 years weekly returns
  - Little impact from using a Bayesian risk model

- MSCI World index investment universe
First Generation approaches to Low Volatility Equity investing


<table>
<thead>
<tr>
<th></th>
<th>Mkt</th>
<th>EW</th>
<th>ERB</th>
<th>ERC</th>
<th>MV long only</th>
<th>MD long only</th>
<th>MV</th>
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</thead>
<tbody>
<tr>
<td>Excess return over RF</td>
<td>2.1%</td>
<td>5.7%</td>
<td>5.9%</td>
<td>5.6%</td>
<td>5.2%</td>
<td>4.8%</td>
<td>6.3%</td>
<td>6.2%</td>
</tr>
<tr>
<td>Volatility</td>
<td>18.1%</td>
<td>18.2%</td>
<td>16.5%</td>
<td>14.8%</td>
<td>9.9%</td>
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<tr>
<td>Sharpe ratio</td>
<td>0.12</td>
<td>0.31</td>
<td>0.36</td>
<td>0.38</td>
<td>0.52</td>
<td>0.41</td>
<td>0.70</td>
<td>0.58</td>
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- All strategies out-perform the Market Cap index
- All but EW are defensive with lower volatility
- Sharpe ratios much higher than for Market Cap.
First Generation approaches to Low Volatility Equity investing

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<tr>
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<td></td>
<td></td>
<td></td>
<td>3.6%</td>
<td>3.8%</td>
<td>3.5%</td>
<td>3.1%</td>
<td>2.6%</td>
</tr>
<tr>
<td>Tracking error</td>
<td></td>
<td></td>
<td></td>
<td>5.1%</td>
<td>5.4%</td>
<td>6.6%</td>
<td>13.1%</td>
<td>12.1%</td>
</tr>
<tr>
<td>Information ratio</td>
<td></td>
<td></td>
<td></td>
<td>0.70</td>
<td>0.70</td>
<td>0.52</td>
<td>0.23</td>
<td>0.22</td>
</tr>
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- Positive excess returns for all. MV long-short out-performs the most.
- TE ~ 5% for EW, ERB and ERC and extremely high for MV and MD
- Information ratio higher for EW, ERB and ERC
First Generation approaches to Low Volatility Equity investing

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<td>15.2%</td>
<td>14.5%</td>
<td></td>
</tr>
<tr>
<td>Information ratio</td>
<td>0.70</td>
<td>0.70</td>
<td>0.52</td>
<td>0.23</td>
<td>0.22</td>
<td>0.28</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td>Beta</td>
<td></td>
<td>0.96</td>
<td>0.87</td>
<td>0.81</td>
<td>0.39</td>
<td>0.48</td>
<td>0.27</td>
<td>0.36</td>
</tr>
<tr>
<td>Maximum Drawdown</td>
<td>-56%</td>
<td>-58%</td>
<td>-55%</td>
<td>-53%</td>
<td>-29%</td>
<td>-39%</td>
<td>-22%</td>
<td>-31%</td>
</tr>
<tr>
<td>Annual turnover</td>
<td>10%</td>
<td>39%</td>
<td>37%</td>
<td>58%</td>
<td>151%</td>
<td>162%</td>
<td>220%</td>
<td>296%</td>
</tr>
</tbody>
</table>


- Defensive beta, in particular for MV and MD
- Drawdown in line with volatility, larger for riskier strategies
- Turnover is larger for MV and MD, and much larger if not constrained long-only
First Generation approaches to Low Volatility Equity investing

Correlation of excess returns: MSCI World index universe

- Weekly data, Jan-97 - Dec-10

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<th>MV</th>
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</tr>
</thead>
<tbody>
<tr>
<td>EW</td>
<td>100%</td>
<td>88%</td>
<td>79%</td>
<td>25%</td>
<td>33%</td>
<td>17%</td>
<td>30%</td>
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<tr>
<td>ERB</td>
<td>100%</td>
<td>97%</td>
<td></td>
<td>57%</td>
<td>61%</td>
<td>52%</td>
<td>56%</td>
</tr>
<tr>
<td>ERC</td>
<td>100%</td>
<td></td>
<td></td>
<td>71%</td>
<td>75%</td>
<td>66%</td>
<td>71%</td>
</tr>
<tr>
<td>MV long only</td>
<td></td>
<td></td>
<td></td>
<td>100%</td>
<td>96%</td>
<td>94%</td>
<td>92%</td>
</tr>
<tr>
<td>MD long only</td>
<td></td>
<td></td>
<td></td>
<td>100%</td>
<td>88%</td>
<td>92%</td>
<td></td>
</tr>
<tr>
<td>MV</td>
<td></td>
<td></td>
<td></td>
<td>100%</td>
<td></td>
<td>93%</td>
<td></td>
</tr>
<tr>
<td>MD</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>100%</td>
<td></td>
</tr>
</tbody>
</table>


- EW, ERB and ERC highly correlated with each other
- MV and MD highly correlated with each other
- Low correlation between the two groups
First Generation approaches to Low Volatility Equity investing

Overlap analysis: MSCI World index universe

- Weekly data, Jan-97 - Dec-10
  - Average portfolio overlap: sum of the smallest weight of each stock in either portfolio

Average overlap of portfolios in capitalisation weight

<table>
<thead>
<tr>
<th>Market Cap</th>
<th>EW</th>
<th>ERB</th>
<th>ERC</th>
<th>MV long only</th>
<th>MD long only</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market Cap</td>
<td>100%</td>
<td>49%</td>
<td>49%</td>
<td>47%</td>
<td>5%</td>
</tr>
<tr>
<td>EW</td>
<td>100%</td>
<td>88%</td>
<td>83%</td>
<td>6%</td>
<td>7%</td>
</tr>
<tr>
<td>ERB</td>
<td>100%</td>
<td>100%</td>
<td>88%</td>
<td>8%</td>
<td>7%</td>
</tr>
<tr>
<td>ERC</td>
<td>100%</td>
<td></td>
<td>100%</td>
<td>10%</td>
<td>11%</td>
</tr>
<tr>
<td>MV long only</td>
<td></td>
<td></td>
<td></td>
<td>100%</td>
<td>61%</td>
</tr>
<tr>
<td>MD long only</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>100%</td>
</tr>
</tbody>
</table>


- Large overlap among EW, ERB and ERC. Large overlap between MV and MD.

Average number of stocks

<table>
<thead>
<tr>
<th>Market Cap</th>
<th>EW</th>
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<th>MV long only</th>
<th>MD long only</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average number of stocks</td>
<td>1746</td>
<td>1746</td>
<td>1746</td>
<td>1746</td>
<td>111</td>
</tr>
</tbody>
</table>

- EW, ERB and ERC invest in all available stocks. MV and MD are concentrated in only 7% of the universe
First Generation approaches to Low Volatility Equity investing

Extended Fama-French analysis: MSCI World Index universe

- Regression of log excess returns over market cap index
  - Five factor regression: Market Cap, SMB, HML, LBMHB, LRVMHRV
  - Weekly data, Jan-97 - Dec-10

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<tbody>
<tr>
<td>Intercept</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Mkt-Cash</td>
<td>-0.01</td>
<td>-0.12</td>
<td>-0.20</td>
<td>-0.71</td>
<td>-0.63</td>
</tr>
<tr>
<td>SMB</td>
<td>0.37</td>
<td>0.32</td>
<td>0.26</td>
<td>0.03</td>
<td>0.06</td>
</tr>
<tr>
<td>HML</td>
<td>0.14</td>
<td>0.16</td>
<td>0.15</td>
<td>-0.02</td>
<td>-0.04</td>
</tr>
<tr>
<td>LBMHB</td>
<td>0.06</td>
<td>0.16</td>
<td>0.24</td>
<td>0.54</td>
<td>0.57</td>
</tr>
<tr>
<td>LRVMHRV</td>
<td>-0.02</td>
<td>0.09</td>
<td>0.05</td>
<td>0.12</td>
<td>0.04</td>
</tr>
<tr>
<td>R-square</td>
<td>75%</td>
<td>79%</td>
<td>84%</td>
<td>84%</td>
<td>78%</td>
</tr>
</tbody>
</table>


- LBMHB: Low beta relative to high beta stock orthogonal to LVMHV
- LRVMHRV: Low residual volatility relative to high residual volatility stocks orthogonal to LBMH
- LBMHB and LVMHV are beta neutral and short-leg is shrunk so that beta is zero
First Generation approaches to Low Volatility Equity investing

Factor replication of excess returns over Market Cap index: MSCI World index universe

- Factor portfolios based on the sum product of factor performances with factor exposures

Data sources: Exshare®, BNP Paribas Investment Partners and MSCI
First Generation approaches to Low Volatility Equity investing

Analytical Perspective: under a single factor risk model

- Stock beta $\beta_i$  Stock residual volatility $\sigma_{\varepsilon,i}$
- MV long-only

$$w_{MV,i} = \frac{\sigma_{MV}^2}{\sigma_{\varepsilon,i}^2} \left( 1 - \frac{\beta_i}{\beta_L} \right) \quad \text{for} \quad \beta_i < \beta_L \quad \text{else} = 0$$

Stocks included only if $\beta_i$ is larger than threshold $\beta_L$

The lower $\beta_i$ the large the stock weight $w_{MV,i}$

The lower the residual variance $\sigma_{\varepsilon,i}^2$ the larger the weight, but cannot drive stock out of the portfolio

First Generation approaches to Low Volatility Equity investing

Analytical Perspective: under a single factor risk model

- Stock beta $\beta_i$  Stock residual volatility $\sigma_{\epsilon,j}$
- MD long-only

$$w_{MD,i} = \frac{\sigma_{MD}^2}{\sigma_{\epsilon,i}^2} \frac{\sigma_i}{\sigma_A} \left(1 - \frac{\rho_i}{\rho_L}\right)$$

for $\rho_i < \rho_L$ else = 0

$\rho_L = \text{long-only threshold beta}$

$\sigma_{MD} = \text{risk of the minimum variance portfolio}$

$\sigma_A = \text{weighted average of volatility of assets in portfolio}$

Stocks included only if $\rho_i$ is larger than threshold $\rho_L$

The lower $\rho_i$ the large the stock weight $w_{MD,i}$

The lower the stock volatility the larger the weight as opposed to stock variance in MD

First Generation approaches to Low Volatility Equity investing

Analytical Perspective: under a single factor risk model

- Stock beta $\beta_i$  
  Stock residual volatility $\sigma_{\varepsilon,i}$

$$w_{ERC,i} = \frac{\sigma^2_{ERC}}{\sigma^2_{\varepsilon,i}} \left[ \left( \frac{\beta_i^2}{\gamma^2} + \frac{1}{N} \frac{\sigma^2_{\varepsilon,i}}{\sigma^2_{ERC}} \right)^{1/2} - \frac{\beta_i}{\gamma} \right]$$

$\gamma = \text{constant}$

$N = \text{number of assets}$

$\sigma_{ERC} = \text{risk of the equal-risk contribution portfolio}$

All stocks are included in the ERC portfolio – long-only by definition

The lower the residual variance $\sigma^2_{\varepsilon,i}$, the larger the stock weight

Stock weights tend asymptotically to zero with higher stock beta $\beta_i$

- and asymptotically to a negatively sloped line for lower stock beta $\beta_i$

First generation approaches are sub-optimal
Fully explained by exposure to well-know mis-pricings

- Volatility anomaly drives TE and excess returns!
  - EW, ERB and ERC well explained by exposure to smaller-cap stocks and low risk stocks!
  - MV and MD are very similar to each other, and well explained by exposure to low risk stocks!

**But none of these strategies was designed with**
the objective of exploiting the volatility anomaly!

- Pitfalls of First Generation low volatility strategies:
  - MV and MD, very similar, are over-invested in defensive sectors
  - MV and MD are over-sensitive to noise in the risk model: constraining is not optimal.
  - Constraints applied to MV and MD bring in riskier stocks
  - EW, ERB and ERC over-exposed to smaller cap stocks
  - No control of the risk exposure to the risk anomaly
  - No control of how defensive the strategy is or of its TE
Second Generation approaches to Low Volatility Equity investing

Correcting the pitfalls

- Second Generation approaches to Low Volatility Equity investing should:
  
  1. Focus on the volatility anomaly and efficient strategies to profit from it
  
  2. Remain simple and transparent, removing all unnecessary complexity
  
  3. Be customisable to take into account investor constraints: TE, beta, etc.
Second Generation approaches to Low Volatility Equity investing

The Micro and Macro of Low Volatility

- High alpha diversification gains from investing in low volatility stocks from all sectors


- Global universe, developed countries, Dec 96 – Dec 11

- The alpha from low risk stocks in cyclical sectors adds substantial diversification
  - Hardly exploited by first generation approaches
Innovative Low Volatility Strategies

Invest only in low volatility stocks & take into account investors’ constraints

- Investable Universe
- Screen Low Risk Stocks
- Low Risk Stocks only
- Investor Constraints
  1. Long-only constraint
  2. Max individual stock weight constraints
  3. TE constraints
  4. Turnover constraints
- Customised Portfolio
Innovative Low Volatility Strategies

---

Invest **only** in low volatility stocks & take into account investors’ constraints

### Profiting from the volatility anomaly

#### Strategy 1: BNP P L1 Equity World Low Volatility

- Low volatility stock portfolio with low TE
  - Screen low volatility stocks
  - Control TE ~ 5% to 6%

#### Strategy 2: BNP P L1 Equity World Pure Low Volatility

- Well diversified portfolio of low volatility stocks
  - Screen low volatility stocks
  - Build well-diversified portfolio of low volatility stocks only

#### Strategy 3: Leveraged Pure Low Volatility strategy

- Leveraged diversified portfolio of low volatility stocks
  - Lower TE relative to market cap by taking leverage
  - Invests on average in 130% of BNP P L1 Pure Low Volatility
  - Leverage financed by borrowing at cash rates

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### Three investment solutions for different type of investors

- **Downside protection:** BNP P L1 Equity World Pure Low Volatility fund & BNP P L1 Equity World Low Volatility fund
- **Peer risk and TE constraints:** BNP P L1 Equity World Low Volatility fund
- **Increase Sharpe Ratio:** Leveraged Pure Low Volatility strategy & BNP P L1 Equity World Pure Low Volatility Fund
- **Performance Seeking:** Leveraged Pure Low Volatility strategy
Innovative Low Volatility Strategies

Most suitable low volatility solution depends on investor constraints and objectives

<table>
<thead>
<tr>
<th>Jan 1995 - Dec 2011</th>
<th>Total Return (%)</th>
<th>Volatility (%)</th>
<th>Excess Returns (%)</th>
<th>Tracking Error (%)</th>
<th>Sharpe Ratio</th>
<th>Information Ratio</th>
<th>Objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSCI World Index</td>
<td>5.6</td>
<td>16.0</td>
<td></td>
<td>0.09</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MSCI Minimum Volatility Index</td>
<td>7.9</td>
<td>11.4</td>
<td>2.2</td>
<td>7.3</td>
<td>0.36</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td>BNP Paribas L1 Equity World Low Volatility</td>
<td>9.6</td>
<td>13.1</td>
<td>4.0</td>
<td>5.5</td>
<td>0.45</td>
<td>0.72</td>
<td>Improve Sharpe ratio. Low TE against market cap. No Leverage.</td>
</tr>
<tr>
<td>BNP Paribas L1 Equity World Pure Low Volatility</td>
<td>12.3</td>
<td>10.4</td>
<td>6.6</td>
<td>9.1</td>
<td>0.82</td>
<td>0.73</td>
<td>Improve Sharpe ratio. No leverage.</td>
</tr>
<tr>
<td>Leverage Pure Low Volatility</td>
<td>15.0</td>
<td>13.5</td>
<td>9.4</td>
<td>8.5</td>
<td>0.84</td>
<td>1.11</td>
<td>Improve Sharpe ratio and boost returns with leverage.</td>
</tr>
</tbody>
</table>

Hypothetical performance results have many inherent limitations and have been obtained with the benefit of hindsight.

Past performance is not indicative of future results.

Datasource: BNP Paribas IP, MSCI, Exshare®
The alpha and beta of low volatility strategies

Low volatility strategies have low beta and large alpha

- Combination of low beta and high alpha creates option-like pay-off
  - Defends the portfolio in bear markets with low beta and positive alpha
  - Keeps up with market cap portfolio in bull markets for as long as positive alpha covers the beta drag
  - Under-performs in strong bull markets as positive alpha is not large enough to cover the beta drag

BNP P L1 Equity World Low Volatility Fund

Regression Analysis (Jan-95 through Dec-11)

\[ R_{\text{fund}} - R_{\text{cash}} \text{ against } R_{\text{MSCI World Index}} - R_{\text{cash}} \]

<table>
<thead>
<tr>
<th>Slope</th>
<th>Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beta</td>
<td>Alpha Jensen (monthly)</td>
</tr>
<tr>
<td>0.78</td>
<td>0.33%</td>
</tr>
<tr>
<td></td>
<td>Alpha Jensen (annualised)</td>
</tr>
<tr>
<td></td>
<td>4.05%</td>
</tr>
</tbody>
</table>

BNP P L1 Equity World Pure Low Volatility Fund

Regression Analysis (Jan-95 through Dec-11)

\[ R_{\text{fund}} - R_{\text{cash}} \text{ against } R_{\text{MSCI World Index}} - R_{\text{cash}} \]

<table>
<thead>
<tr>
<th>Slope</th>
<th>Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beta</td>
<td>Alpha Jensen (monthly)</td>
</tr>
<tr>
<td>0.55</td>
<td>0.56%</td>
</tr>
<tr>
<td></td>
<td>Alpha Jensen (annualised)</td>
</tr>
<tr>
<td></td>
<td>6.99%</td>
</tr>
</tbody>
</table>

- Generates higher Sharpe ratio than market cap because of positive alpha

Datasource: BNP Paribas IP, MSCI, Exshare®
Beta exposures
Higher beta with lower Tracking Error

- Higher beta for the low TE strategy (BNP P L1 Equity World Low volatility Fund).
- Lower beta for the Pure Low Volatility Strategy, but higher TE

Datasource: BNP Paribas IP, MSCI, Exshare®
Despite its defensive character the strategy used in the BNP L1 Equity World Low Volatility Fund manages to generate a limited beta drag, i.e. negative contribution from defensive beta to overall excess returns.

The beta drag in the Pure Low Volatility Strategy is much larger in line with a much more defensive beta and much higher tracking error risk.

Datasource: BNP Paribas IP, MSCI, Exshare®.
Alpha contribution to excess returns
Cumulated contributions to excess returns over the MSCI World Index

- The BNP P L1 Equity World Low Volatility Fund generates significant alpha
- The Pure Low Volatility Strategy, with much higher TE risk against the MSCI World Index, but also much better diversified in terms of exposure to the alpha from low volatility stocks delivers much more alpha

Datasource: BNP Paribas IP, MSCI, Exshare®
No style biases other than low risk
On average the portfolio has no strong biases in any style other than low risk

- Strong biases imply exposures in excess of +/- 0.8
- No strong biases neither against the universe or against the market capitalisation portfolio

Datasource: BNP Paribas Investment Partners, MSCI, Worldscope, Exshares, IBES.
Market Impact analysis
Low volatility investing strategies tend to have large capacity

- Proprietary model for total costs as a function of assets-under-management (AUM)
  - Expected total cost includes
    - Fixed cost of 4bp (commissions, brokerage fees and taxes)
    - Market impact estimated from a function of bid-ask spread, trade period, stock volatility and participation ratio
  - Model calibrated using historical real market data provided

<table>
<thead>
<tr>
<th>Estimated Annual Total Cost from Market Impact and Fixed Costs</th>
<th>BNP P L1 Equity World Low Volatility</th>
<th>BNP P L1 Equity World Pure Low Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUM on 31-Dec-2010</td>
<td>Implementation over</td>
<td>Implementation over</td>
</tr>
<tr>
<td>USD Million</td>
<td>1 day</td>
<td>5 days</td>
</tr>
<tr>
<td>500</td>
<td>0.33%</td>
<td>0.23%</td>
</tr>
<tr>
<td>1000</td>
<td>0.40%</td>
<td>0.27%</td>
</tr>
<tr>
<td>2000</td>
<td>0.51%</td>
<td>0.32%</td>
</tr>
<tr>
<td>5000</td>
<td>0.70%</td>
<td>0.42%</td>
</tr>
</tbody>
</table>

- Strategies have low alpha decay.
  - Implementation can be spread over 5 days without lowering excess returns

Datasource: BNP Paribas IP, MSCI, Exshare®
Conclusions

Benefiting from the alpha from low volatility stocks

- Low risk stocks have generated higher returns than higher risk stocks
  - Globally and across regions and sectors.
  - This anomaly can be explained by investor behaviour

- This anomaly explains the behaviour of First Generation risk-based strategies such as Minimum Variance, Maximum Diversification, Equal-Risk Contribution and Equal Risk-Budgeting.

- Second Generation low volatility strategies will invest only in low risk stocks and take into account investor constraints, e.g. tracking error constraints.

- If low tracking error is not a constraint, Second Generation solutions still much more attractive than First Generation risk based strategies.

- When investors target performance, leverage can be used to increase exposure to the alpha of low volatility stocks.
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